THERMAL PROCESSES BETWEEN A RIGID WALL AND A STREAM OF HOT GAS

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UDC 536.2.083

Systems of linearized heat-conduction equations with appropriate initial and boundary conditions are derived. Based on the solutions of these equations, a method is presented for determining intense heat fluxes incident on surfaces of bodies of finite thickness.

In the design of modern steam generators and shielding articles located in regions of very large thermal stresses, centralized heat supply systems and central heating systems (atomic power plants, heat and electric power plants, network substations, heat supply systems), etc., the problem of determining both steady and unsteady heat fluxes incident on the surfaces of individual structures is crucial. A more accurate solution of this problem, i.e., a solution of the nonlinear heat-conduction equation with appropriate boundary conditions, leads to decreased weights and safety factors of individual units and, consequently, to increased economic efficiency of equipment manufacture.

Currently, three groups of methods for solving nonlinear equations can be distinguished: 1) analytic; 2) numerical; 3) mathematical modeling.

When analytic solutions can be obtained they are to be preferred when they are simple and can be evaluated with a minimum expenditure of working time.

We present an analytic method for solving nonlinear heat-conduction equations which to a certain extent meets these requirements.

Suppose it is required to solve the nonlinear heat-conduction equation

$$\rho_0 (C_0 + C_1\Theta) \frac{\partial \Theta}{\partial \tau} = \frac{\partial}{\partial x}\left((\lambda_0 + \lambda_1 \Theta) \frac{\partial \Theta}{\partial x} \right) \ (0 < x < R_2,\ \tau > 0) \tag{1}$$

with boundary conditions of the form

$$\Theta|_{\tau=0} = 0 \quad (R_1 \leqslant x \leqslant R_2), \tag{2}$$

$$\Theta|_{x=R_1} = \varphi_1(\tau) \ (\tau > 0), \tag{3}$$

$$\Theta|_{x=R_2} = \varphi_2(\tau) \ (\tau > 0), \tag{4}$$

where $\Theta = t - t_0$, R_1 is the distance from the point $x = 0$, R_2 is the thickness of the plate, and

$$\lambda(\Theta) = \lambda_0 + \lambda_1 \Theta, \tag{5}$$

$$C(\Theta) = C_0 + C_1 \Theta \tag{6}$$

give the temperature dependence of the thermal conductivity and the specific heat, respectively.

If the functions (5) and (6) are inserted under the appropriate derivative signs Eqs. (1)-(4) can be written as

$$\frac{\partial}{\partial \tau}\left(\Theta + \frac{C_1}{2C_0}\Theta^2 \right) = a_0 \frac{\partial^2}{\partial x^2}\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2 \right), \tag{7}$$

$$\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2 \right)\Big|_{\tau=0} = 0, \tag{8}$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 3, pp. 511-516, September, 1977. Original article submitted August 31, 1976.

0022-0841/77/3303-1097$07.50

$$\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right)\Big|_{x=R_1} = \varphi_1(\tau) + \frac{\lambda_1}{2\lambda_0}\varphi_1^2(\tau) = \psi_1(\tau), \tag{9}$$

$$\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right)\Big|_{x=R_2} = \varphi_2(\tau) + \frac{\lambda_1}{2\lambda_0}\varphi_2^2(\tau) = \psi_2(\tau). \tag{10}$$

The functions

$$\varphi(\Theta) = \Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2, \tag{11}$$

$$\psi(\Theta) = \Theta + \frac{C_1}{2C_0}\Theta^2 \tag{12}$$

are continuous and differentiable, satisfy the Dirichlet conditions [1], and can be expanded in Fourier series in the interval $(0, \Theta_p)$

$$\varphi(\Theta) = \sum_{k=1}^{\infty} b_k \sin\frac{k\pi\Theta}{\Theta_p}, \tag{13}$$

$$\psi(\Theta) = \sum_{k=1}^{\infty} B_k \sin\frac{k\pi\Theta}{\Theta_p}, \tag{14}$$

$$b_k = \frac{2}{\Theta_p}\int_0^{\Theta_p} \varphi(\Theta)\sin\frac{k\pi\Theta}{\Theta_p}\,d\Theta, \tag{15}$$

$$B_k = \frac{2}{\Theta_p}\int_0^{\Theta_p} \psi(\Theta)\sin\frac{k\pi\Theta}{\Theta_p}\,d\Theta. \tag{16}$$

Substituting Eqs. (13) and (14) into (7) and formally applying the reduction rule [2] reduces Eqs. (7)-(10) to the form

$$\frac{\partial T_k}{\partial\tau} = a_0\alpha_k\frac{\partial^2 T_k}{\partial x^2}, \tag{17}$$

$$T_k|_{\tau=0} = 0, \tag{18}$$

$$T_k|_{x=R_1} = \frac{1}{e(k-1)!}\psi_1(\tau), \tag{19}$$

$$T_k|_{x=R_2} = \frac{1}{e(k-1)!}\psi_2(\tau), \tag{20}$$

where

$$T_k = b_k \sin\frac{k\pi\Theta}{\Theta_\Pi}, \tag{21}$$

$$\alpha_k = \frac{b_k}{B_k} = \frac{\cos k\pi + \frac{\lambda_1}{2\lambda_0}\Theta_\Pi\left[\left(1 - \frac{2}{(k\pi)^2}\right)\cos k\pi + \frac{2}{(k\pi)^2}\right]}{\cos k\pi + \frac{C_1}{2C_0}\Theta_\Pi\left[\left(1 - \frac{2}{(k\pi)^2}\right)\cos k\pi + \frac{2}{(k\pi)^2}\right]}, \tag{22}$$

and e is the base of natural logarithms.

TABLE 1. Values of Coefficients α_k (Θ_p = 600°C)

k	α_k	k	α_k	k	α_k	k	α_k
1	0,93434	6	0,89236	11	0,89271	16	0,89236
2	0,89236	7	0,89314	12	0,89236	17	0,89245
3	0,89694	8	0,89236	13	0,89262	18	0,89236
4	0,89236	9	0,89289	14	0,89236	19	0,89244
5	0,89394	10	0,89236	15	0,89253	20	0,89236

TABLE 2. Temperature Distribution of a Plate as a Function of Time and Position, °C

τ, sec	R, mm			
	1	2	3	5
0,01	4,5312	0,9120	0,1376	0,0013
0,02	17,6043	6,2493	1,9182	0,1134
0,03	34,9976	15,6937	6,3866	0,3752
0,04	54,3102	27,8479	13,2755	2,3903
0,05	74,2323	41,6127	22,0037	5,1075
0,06	94,0390	56,2270	32,0196	8,8931
0,07	113,3391	71,1849	42,8785	13,6243
0,08	131,9357	86,1590	54,2441	19,1476
0,09	149,7466	100,9440	65,8712	25,3092
0,10	166,7560	115,4163	77,5854	31,9698
0,11	182,9859	129,5073	89,2654	39,0094
0,12	198,4782	143,1841	100,8293	46,3280
0,13	213,2840	156,4372	112,2233	53,8442
0,14	227,4578	169,2715	123,4142	61,4928
0,15	241,0525	181,7010	134,3831	69,2221
0,16	254,1187	193,7444	145,1210	76,9915
0,17	266,7029	205,4229	155,6259	84,7697
0,18	278,8475	216,7586	165,9003	92,5324
0,19	290,5906	227,7734	175,9498	100,2612
0,20	301,9664	238,4881	185,7819	107,9425
0,21	313,0051	248,9227	195,4051	115,6659
0,22	323,7338	259,0956	204,8282	123,1240
0,23	334,1767	269,0239	214,0606	130,6117
0,24	344,3551	278,7232	223,1112	138,0254
0,25	354,2881	288,2079	231,9887	145,3629
0,26	363,9927	297,4912	240,7017	152,6230
0,27	373,4843	306,5852	249,2579	159,8053
0,28	382,7765	315,5009	257,6650	166,9099
0,29	391,8817	324,2481	265,9300	173,9374
0,30	400,8110	332,8363	274,0595	180,8886

Thus, we obtain a system of linear equations which is readily solved. Taking the Laplace transform, the expression for the transform of T_k is

$$\bar{T}_k = \frac{1}{e(k-1)!}\left[\psi_1(s)\,\frac{\operatorname{sh}\sqrt{\frac{s}{a_0\alpha_k}}\,(R_2-x)}{\operatorname{sh}\sqrt{\frac{s}{a_0\alpha_k}}\,(R_2-R_1)} + \psi_2(s)\,\frac{\operatorname{sh}\sqrt{\frac{s}{a_0\alpha_k}}\,(x-R_1)}{\operatorname{sh}\sqrt{\frac{s}{a_0\alpha_k}}\,(R_2-R_1)}\right], \tag{23}$$

and after taking the inverse transform and differentiating with respect to x, we find

$$\left.\frac{\partial T_k}{\partial x}\right|_{x=0} = -\frac{1}{e(k-1)!}\left(\psi_1(\tau)-\psi_2(\tau)\right)\frac{1}{R_2-R_1} - \frac{1}{e(k-1)!}\,\psi_1'(\tau)\,\frac{2R_2^2+2R_2R_1-R_1^2}{6a_0\alpha_k(R_2-R_1)} - \frac{1}{e(k-1)!}\,\psi_2'(\tau)\,\frac{R_2^2-2R_2R_1-2R_1^2}{6a_0\alpha_k(R_2-R_1)}, \tag{24}$$

if we limit ourselves to first-order derivatives of $\psi_1(\tau)$ and $\psi_2(\tau)$. It should be noted that taking account of second derivatives of these functions does not change the result significantly since in a short time interval Δτ these functions are commonly taken as linear.

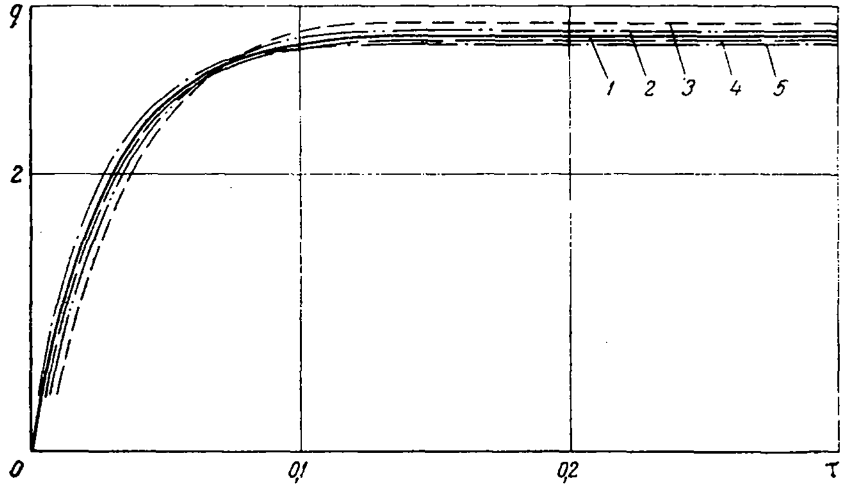

Fig. 1. Results of reconstruction of heat fluxes: curve 1 by Eq. (29); 2, 3, 4, 5 by (27) with R_1 = 1 mm and R_2 = 5 mm; R_1 = 2 mm and R_2 = 5 mm; R_1 = 1 mm and R_2 = 3 mm; R_1 = 3 mm and R_2 = 5 mm, respectively. q, kW/cm²; τ, sec.

In most experimental studies, detectors for sensing heat fluxes are made of copper, for which Eqs. (5) and (6) have the form

$$\lambda(\Theta) = 390 - 0.0617\Theta, \tag{25}$$

$$c(\Theta) = 387 + 0.0870\Theta. \tag{26}$$

Using the data in Table 1 the values of α_k for Θ_p = 600°C were determined. This value of Θ_p was taken as the maximum temperature at any point x in the time interval under consideration. This ensures the convergence of the Fourier series to the functions expanded. Table 1 shows that α_1 = 0.93434 differs from all the other values of α_k by about 5%, and they in turn differ so slightly from one another that their average α_k = 0.89280 can be taken for all k > 1. This procedure eliminates the summation over k and makes it possible to write the expression for the heat fluxes in the form

$$q(\tau) = \lambda_0 \left[(\psi_1(\tau) - \psi_2(\tau)) \frac{1}{R_2 - R_1} + \right.$$
$$+ \psi_1'(\tau) \frac{2R_2^2 + 2R_2R_1 - R_1^2}{6a_0 e (R_2 - R_1)} \left(\frac{1}{\alpha_1} + \frac{e-1}{\alpha_k} \right) +$$
$$\left. + \psi_2'(\tau) \frac{R_2^2 - 2R_2R_1 - 2R_1^2}{6a_0 e (R_2 - R_1)} \left(\frac{1}{\alpha_1} + \frac{e-1}{\alpha_k} \right) \right]. \tag{27}$$

The calculation of heat fluxes by Eq. (27) requires the values of the functions $\psi_1(\tau)$, $\psi_2(\tau)$, $\psi_1'(\tau)$, and $\psi_2'(\tau)$, which are ordinarily taken from experiment. However, another method can be used. In the present paper we solve the nonlinear heat-conduction equation (1) numerically with boundary conditions of the form

$$\Theta|_{\tau=0} = 0, \tag{28}$$

$$(\lambda_0 + \lambda_1\Theta) \frac{\partial\Theta}{\partial x}\Big|_{x=0} = -q_0(1 - e^{-\delta\tau}), \tag{29}$$

$$\Theta|_{x=R} = 0. \tag{30}$$

In Eq. (29) we set $q_0 = 3 \cdot 10^7$ W/m² and $\delta = 31.54$ sec⁻¹, which corresponds most closely to experimental conditions [3].

Table 2 gives the calculated temperature distribution for a copper plate of thickness R = 50 mm. By using this table, boundary conditions (3) and (4) can be chosen for various values of R_1 and R_2 to solve the inverse problem of determining heat fluxes. If the results obtained by Eq. (27) are close to the conditions (29), the proposed method is accurate and can be used to calculate heat fluxes.

Figure 1 shows the results of such a reconstruction of heat fluxes using Eq. (27). Curve 4, calculated for $R_1 = 1$ mm and $R_2 = 3$ mm, measured from the surface of the plate $x = 0$, practically coincides with the reference curve 1 constructed by using Eq. (29). The agreement at early times is somewhat worse for curves 2, 3, and 5 calculated with $R_1 = 1$ mm and $R_2 = 5$ mm, $R_1 = 2$ mm and $R_2 = 5$ mm, and $R_1 = 3$ mm and $R_2 = 5$ mm, respectively. It is clear that this can account for the less accurate approximation of the temperature distribution at $x = 0$. For $\tau > 0.1$ sec, however, all the results are close, and the proposed method of calculating heat fluxes can be used in practice.

NOTATION

ρ, density, kg/m^3; C, specific heat, $J/kg \cdot {}^\circ C$; τ, time, sec; λ, thermal conductivity, $W/m \cdot {}^\circ C$; x, running coordinate, m; t, temperature, °C; t_0, initial temperature, °C; a_0, thermal diffusivity, m^2/sec; q, heat flux, W/m^2.

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PROPAGATION OF HEAT WITH A VARIABLE RELAXATION PERIOD

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UDC 532.24.02

We present an exact solution of the hyperbolic heat-conduction equation for a variable velocity of heat transport.

According to the hypothesis of the finite velocity of heat transport developed by Lykov [1] we have a hyperbolic heat-conduction equation

$$t_r \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} , \tag{1}$$

where t_r is the relaxation period in hours, a^2 is the thermal diffusivity, and $w_q = \sqrt{a^2/t_r}$ is the velocity of propagation of heat.

If t_r and a^2 are constants, w_q is a finite velocity. Under these assumptions we solve certain problems related to Eq. (1) which can be found in [2-4].

Norwood [5] investigated variable values of t_r, and Samarskii and Sobol' [6] used a computer to study temperature waves.

We assume that t_r varies linearly with the time. This case leads to an exact solution of Eq. (1) for many boundary-value problems.

We set

$$t_r = 2t + b, \tag{2}$$

where b is a positive constant. Then the substitution $\xi^2 = 2t + b$ reduces Eq. (1) to the familiar form

Lenin Komsomol Voronezh State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 3, pp. 517-518, September, 1977. Original article submitted October 8, 1976.